MAC Algorithms:
State of the Art and Recent Developments

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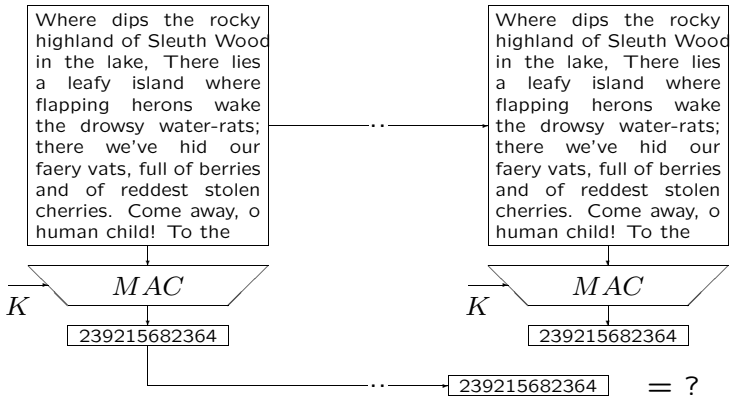
MAC Algorithms: overview

1. definition
2. applications
3. attacks
4. constructions
   - based on block ciphers
   - based on hash functions
   - dedicated MACs
   - based on authentication codes/universal hash functions

MAC = hash function with secret key



MAC: definition (1)

Message Authentication Code

= hash function with secret key:

1. description of h public
2. X arbitrary length $\Rightarrow$ fixed length m (32 . . . 160 bits)
3. computation of $h_K(X)$ "easy" given X and K
4. computation of $h_K(X)$ "hard" given only X, even if a large number of pairs $\{X_i, h_K(X_i)\}$ is known

calculation of $h_K(X)$ without knowledge of secret key:
forgery

- verifiable or not verifiable
- selective or existential

MAC: definition (2)

A MAC is secure if, for an adversary who does not know K, it is computationally infeasible to perform an existential forgery under an adaptive chosen text attack.

A MAC is (ϵ, t, q, q', L) secure if, an adversary who does not know K, and

- can spend time t (operations),
- can obtain the MAC for q texts of this choice,
- can observe the MAC for q' texts (not of his choice),
- and can obtain the result of q'' verification queries on text-MAC pairs of his choice.

(each message of length L),
cannot produce an existential forgery with probability of success larger than ϵ.

Applications

MAC versus digital signature:

− non-repudiation

− key management

\+ performance/computational effort

\+ size of MAC and of keys

- banking
- Internet security: IP security
- electronic purses + authorization for credit cards

Attack: exhaustive key search

try all values of the key K

- $\#X, h_K(X)$ pairs $\approx k/m$
- # attempts $\approx 2^{k-1}$
- a recovered key is only valuable within its lifetime
- but allows for arbitrary forgery

long term security: 75. . . 90 bits

Attack complexity: $[2^k, k/m, 0, 0]$
or $[2^{2k/3}, 0, k/m, 0]$ with 2^k offline work

notation
work − known texts − chosen texts − on-line verifications

‘Attack’: guess MAC

success probability $\max(1/2^m, 1/2^k)$

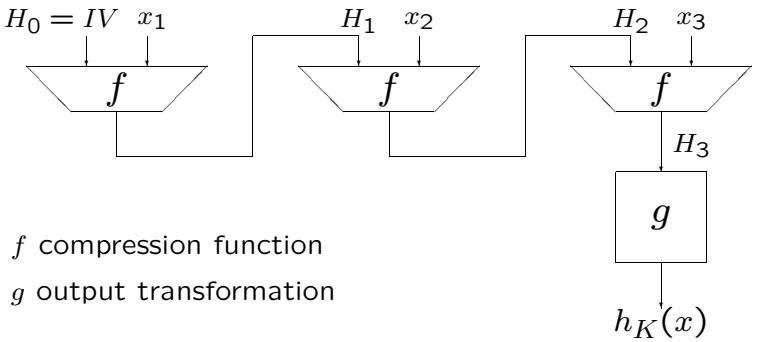
but:

- not verifiable
- requires on-line verification

depending on application: $m, k \geq 32 \ldots 64$

Attack complexity: $[0, 0, 0, \min(2^m, 2^k)]$

Attack: birthday forgery attack (1)



f compression function

g output transformation

unambiguous padding of input to multiple of block length
divide input into blocks x_1, x_2,, . . . , x_t

Birthday forgery attack (2)

find (x, x'): $h(x) = g(H_t) = h(x') = g(H'_t)$

internal collision: $H_t = H'_t$

external collision: $H_t \neq H'_t$ but $g(H_t) = g(H'_t)$

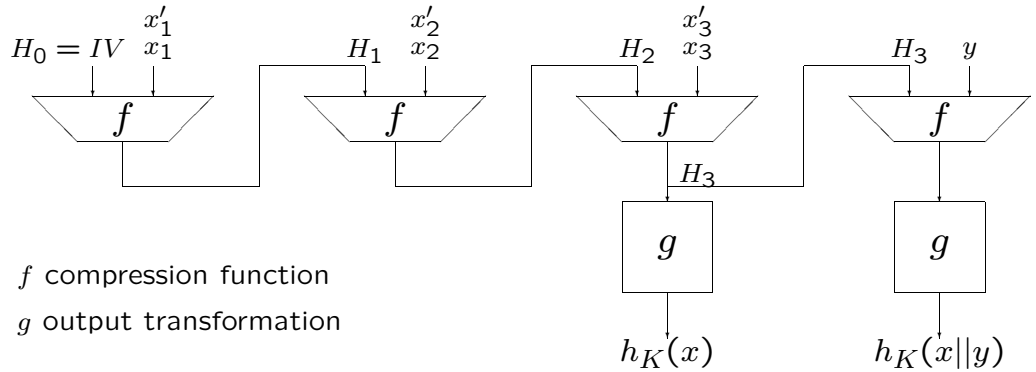
note: if g is bijective, there are no external collisions

Lemma 1 *An internal collision for an iterated MAC can be used for a forgery which requires only a single chosen text.*

indeed: $h_K(x \,\|\, y) = h_K(x' \,\|\, y)$
and thus one can predict the 2nd MAC without knowing K

note: this does NOT work for an external collision.

Attack: birthday forgery attack (2b)



f compression function

g output transformation

unambiguous padding of input to multiple of block length
divide input into blocks x_1, x_2,, . . . , x_t

Birthday forgery attack (3)

- internal memory (H_i): n bits
- MAC value: m bits

[Preneel-van Oorschot] forgery: $[0, 2^{n/2}, \leq 2^{n-m}, 0]$:

- internal collision after $2^{n/2}$ text-MAC pairs
  (by the birthday paradox)
- # external collisions is equal to $(2^{n/2})^2/2^{m+1} = 2^{n-m-1}$
  (or 0 if g is a permutation)
- distinguish internal/external by simulating the attack of lemma 1
  (fails with high probability for external collisions only; 2 chosen texts per collision)

small improvement: $[0, 2^{n/2}, \min(2^{n/2}, 2^{n-m}), 0]$

Birthday paradox

Given a set with S elements.
Choose r elements at random (with replacements), with $r \ll S$.
Find probability p that there at least two identical elements.

$$q = 1 - p = 1 \cdot \frac{S-1}{S} \cdot \frac{S-2}{S} \cdots \frac{S-(r-1)}{S} = \prod_{k=1}^{r-1} \left(1 - \frac{k}{S}\right)$$

$$\ln q = \sum_{k=1}^{r-1} \ln\left(1 - \frac{k}{S}\right) \approx -\sum_{k=1}^{r-1} \frac{k}{S} = -\frac{r(r-1)}{2S}$$

$$p = 1 - \exp\left(-\frac{r(r-1)}{2S}\right) \quad \text{if } r = \sqrt{S} : p = 39\%$$

birthday paradox: for $S = 365$ and $r = 23$, $p = 1/2$
intuition: number of pairs of elements is $r(r-1)/2$.

Birthday forgery attack (4)

Let h be an iterated MAC with n-bit chaining variable and m-bit result.
An internal collision for h can be found with

$\sqrt{2} \cdot 2^{n/2}$ known texts

v chosen texts.

$v = 0$ if g is a permutation;

$$v = \min\left(2^{n/2}, 2\left(2^{n-m}\left(1 - \frac{1}{e}\right) + \left\lfloor \frac{n-1}{m-1} \right\rfloor\right)\right) \quad \text{otherwise.}$$

Birthday forgery attack (5)

Let h be an iterated MAC with n-bit chaining variable, m-bit result, a compression function f which behaves like a random function (for fixed x_i), and output transformation g.

An internal collision for h can be found using u known text-MAC pairs, where each text has the same substring of $s \geq 0$ trailing blocks, and v chosen texts.

$u = \sqrt{2/(s+1)} \cdot 2^{n/2}$

$v = 0$ if the output transformation g is a permutation or if $s + 1 \geq 2^{n-m+6}$

$$v = 2\left(\frac{2^{n-m}}{s+1} \cdot \left(1 - \frac{1}{e}\right) + \left\lfloor \frac{n - 1 - \log_2(s+1)}{m-1} \right\rfloor\right),$$

Birthday forgery attack (6)

practical?

- not all environments allow for chosen texts
- optimization reduces number of known texts
- extension to key recovery is more dangerous

how to preclude? see later

MAC algorithms based on a block cipher

ISO 9797-1 (2001)

- 6 variants of CBC-MAC
- 3 padding schemes
  - padd with zeroes
  - padd with 1 (always) followed by zeroes
  - prepend length in bits and padd with zeroes

Other schemes

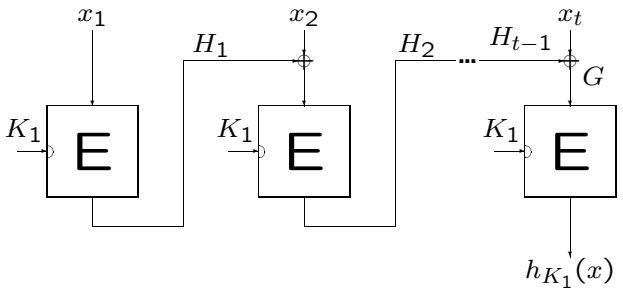
XOR MAC

PMAC

3GPP-MAC

XCBC

CBC-MAC: Algorithm 1 (1)



- ANSI 1982, FIPS 1985, ISO 1987, ISO/IEC 1989
- proof of security for fixed length inputs by [Bellare-Kilian-Rogaway'94]
- $m = 32 \ldots 64$ bits

CBC-MAC: Algorithm 1 (2)

security with DES:

- key search: $[2^{56}, 2, 0, 0]$
- key recovery using Ic: $[2^{43}, 2^{43}, 0, 0]$
- guess MAC: $[0, 0, 0, \min(2^{56}, 2^m)]$
- birthday forgery attack (even if triple-DES):
  - $m = 64$: $[0, 2^{32}, 1, 0]$
  - $m = 32$: $[0, 2^{32}, 2^{33}, 0]$
- trivial forgery for $m = 64$ if no special operation for last block
- improved attack for $m = 32$: $[0, 2, 2^{16}, 0]$ [Knudsen97]

much smaller than expected!

Why a special operation for the last block?

x consists of a single block

- $\mathrm{MAC}_K(x)$ is known
- then $\mathrm{MAC}_K(x \| (x \oplus \mathrm{MAC}_K(x))) = \mathrm{MAC}_K(x)$.

x, x' consist of a single block

- $\mathrm{MAC}_K(x)$ and $\mathrm{MAC}_K(x')$ are known
- $\mathrm{MAC}_K(x \| (x' \oplus \mathrm{MAC}_K(x))) = \mathrm{MAC}_K(x')$.

x, x', and Y fall on block boundaries

- $\mathrm{MAC}_K(x)$, $\mathrm{MAC}_K(x \| Y)$, and $\mathrm{MAC}_K(x')$ are known
- $\mathrm{MAC}_K(x' \| Y') = \mathrm{MAC}_K(x \| Y)$
  if $Y' = Y \oplus \mathrm{MAC}_K(x) \oplus \mathrm{MAC}_K(x')$.

Knudsen's attack for $m < n$ (1)

probabilistic variant of the attack

- 2 known texts: $\mathsf{MAC}_K(x) = \alpha_m$ and $\mathsf{MAC}_K(x') = \alpha'_m$
- output of last encryption $= \alpha_m \| \beta_{n-m}$ and $\alpha'_m \| \beta'_{n-m}$, with α, α' known and β, β' unknown.
- chosen texts: r pairs of the following form: $x \| \alpha_m \| \gamma_{n-m}$ and $x' \| \alpha'_m \| \gamma'_{n-m}$.
- input block for the last encryption (G) is of the form $0_m \| (\beta \oplus \gamma)_{n-m}$ and $0_m \| (\beta' \oplus \gamma')_{n-m}$

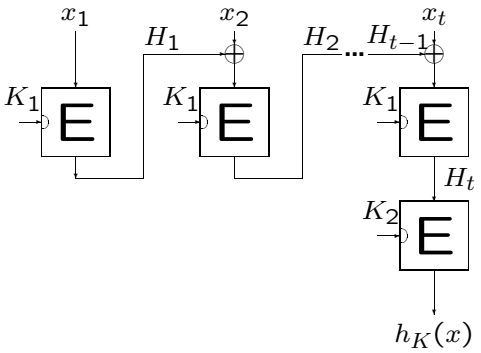
Knudsen's attack for $m < n$ (2)

- if for a pair $\beta \oplus \gamma = \beta' \oplus \gamma'$, the two MACs for that pair will be equal; this is easy to detect.
- this happens with probability 0.63 if $r = \sqrt{2^{n-m}}$
- based on the collision, we can compute $\beta \oplus \beta' := \gamma \oplus \gamma$'
- this leads to a forgery as follows: $\mathsf{MAC}_K(x \| Y) = \mathsf{MAC}_K(x' \| (Y \oplus (\Delta\alpha_m \| \Delta\beta_{n-m}))$ with $\Delta\alpha = \alpha \oplus \alpha'$ and $\Delta\beta = \beta \oplus \beta'$.

2 known texts, $2 \cdot 2^{(n-m)/2} + 1$ chosen texts

CBC-MAC: Algorithm 2 (1)

RIPEMD-MAC [RIPE'93] + EMAC (DMAC) [Petrank-Rackoff'98]



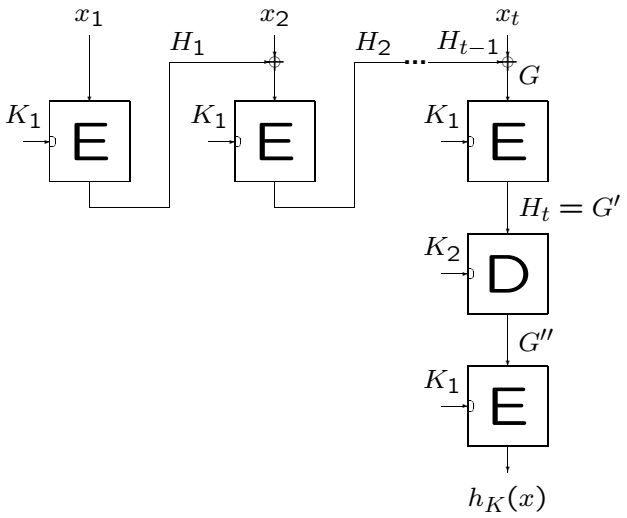
CBC-MAC: Algorithm 2 (2)

security with DES:

- key search: $[2^{56}, 2, 0, 0]$
- guess MAC: $[0, 0, 0, \min(2^{56}, 2^m)]$
- birthday forgery attack (even if triple-DES):
  - $m = 64$: $[0, 2^{32}, 1, 0]$
  - $m = 32$: $[0, 2^{32}, 2^{33}, 0]$

much smaller than expected!

CBC-MAC: Algorithm 3 (retail MAC) (1)



CBC-MAC: Algorithm 3 (retail MAC) (2)

security with DES and $m = 64$:

- key search: $[2^{112}, 2, 0, 0]$
- guess MAC: $[0, 0, 0, \min(2^{112}, 2^m)]$
- birthday forgery attack: $[0, 2^{32}, 1, 0]$ (or $[0, 2^{32}, 2^{33}, 0]$)
- improved key recovery [Preneel-van Oorschot-Knudsen]
  - $[3 \cdot 2^{56}, 2^{32}, 0, 0]$
  - $[3 \cdot 2^{56}, 1, 0, 2^{56}]$

solution: triple-DES in first and last round?

Key recovery attack on retail MAC (1)

- collect 2^{32} known text-MAC pairs (e.g., of 2 blocks)
- with probability 0.39 there is a collision (x, x')
- find the keys K_1 for which the input to the last triple encryption (G) is the same for x and x'
  - with high probability, there will be only 1 solution
  - work factor: $2 \cdot 2^{56}$ encryptions
- use K_1 to compute G' and G''
- find the key K_2: 2^{56} encryptions

$2^{32.5}$ known texts and $3 \cdot 2^{56}$ encryptions

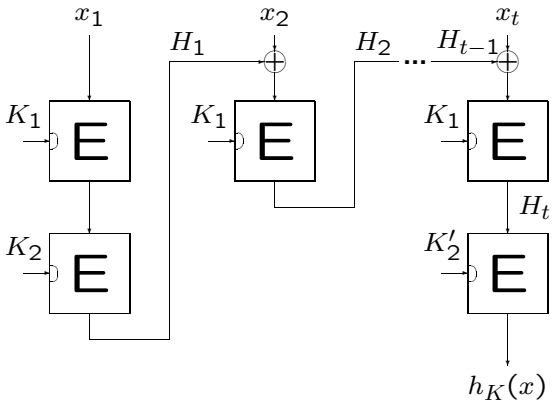
Key recovery attack on retail MAC (2)

- one known text $\alpha = \mathrm{MAC}_K(x)$, say of 2 blocks x_1, x_2
- guess K_1
  - compute the value of G (input of the last triple encryption)
  - choose $x_1' \neq x_1$ and x_2' such that the same value for G is obtained, or $x_2' = E_{K_1}(x_1') \oplus G$
  - ask a MAC verification device: $\mathrm{MAC}_K(x_1' \| x_2') = \alpha$?
  - if yes, the guess for K_1 was right (with high probability)
- use K_1 to compute G' and G''
- find the key K_2: 2^{56} encryptions

1 known text, $3 \cdot 2^{56}$ encryptions and 2^{56} MAC verifications

CBC-MAC: Algorithm 4 (Mac-DES) (1)

[Knudsen-Preneel'98]



CBC-MAC: Algorithm 4 (Mac-DES) (2)

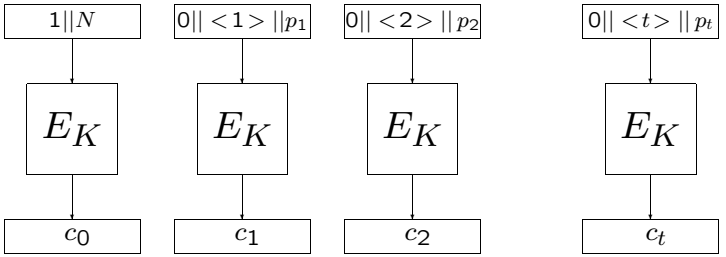
security with DES and $m = 64$:

- key search: $[2^{112}, 2, 0, 0]$
- guess MAC: $[0, 0, 0, \min(2^{112}, 2^m)]$
- birthday forgery attack:
  $[0, 2^{32}, 1, 0]$, for $m = 32$: $[0, 2^{32}, 2^{33}, 0]$
- improved key recovery [Coppersmith-Mitchell-Knudsen00]:
  $[2^{59}, 2^{33}, 3 \cdot 2^{49}, 0]$, for $m = 32$: $[2^{64}, 0, 2^{63}, 2^{57}]$

stronger against key recovery than MAC Algorithm 3

XOR MAC

[Bellare-Guérin-Rogaway'95]



Σ, N with $\Sigma = c_0 \oplus c_1 \oplus c_2 \oplus \cdots c_t$

- stronger security reduction than CBC-MAC (linear decrease of security with number of blocks)
- incremental and parallelizable
- twice as slow as CBC-MAC (for 32-bit length field)
- MAC twice as long

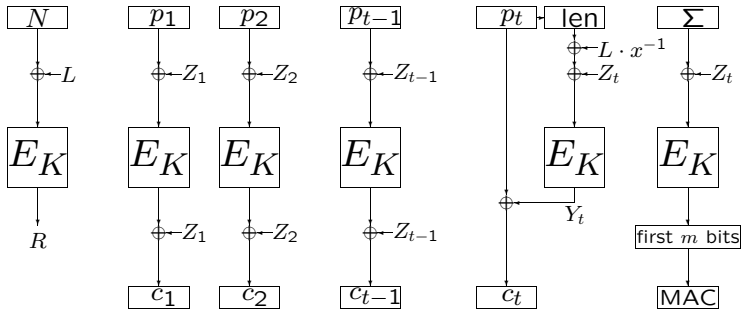
OCB mode and PMAC (1)

[Rogaway-Bellare-Black-Krovetz 01]

- authenticated encryption: indistinguishability under chosen plaintext attack and authenticity of ciphertexts
- randomized encryption using a nonce N: nonce can be used only once but does not need to be unpredictable
- one block cipher key (but 3 keys)
- any input length: no need for multiple of block length
- fully parallel, preprocessing possible
- minimal ciphertext expansion: MAC and nonce
- only two extra block cipher calls

PMAC: variant with only MAC
optimization: [Rogaway04] (Asiacrypt 2004)

OCB mode and PMAC (2)



$\Sigma := p_1 \oplus \ldots \oplus p_{t-1} \oplus c_t||0^* \oplus Y_t$
MAC := first bits of $E_K(\Sigma \oplus Z_t)$

security bound: $O\left(\frac{q'^2}{2^n} + \frac{1}{2^m}\right)$ with q' total number of blocks

OCB mode and PMAC (3)

input: $p_1 ... p_{t-1}\, p_t$ (p_t can be incomplete)

1. $L := E_K(0^n)$ and $R := E_K(N \oplus L)$
2. for $i := 1$ to t do $Z_i := L_i \oplus R$
3. for $i := 1$ to $t-1$ do $c_i := E_K(p_i \oplus Z_i) \oplus Z_i$
4. $Y_t := E_K(\mathsf{len}(p_t) \oplus L_{-1} \oplus Z_t)$
5. $c_t := Y_t \oplus p_t$
6. $\Sigma := p_1 \oplus \ldots \oplus p_{t-1} \oplus c_t||0^* \oplus Y_t$
7. MAC := first bits of $E_K(\Sigma \oplus Z_t)$

$L_1 := L$, $L_i := L_{i-1} \cdot x^{\mathsf{ntz}(i)}$, $L_{-1} := L \cdot x^{-1}$
multiplication in GF(2^{128}) with $f(x) = x^{128} + x^7 + x^2 + x + 1$
ntz(i): number of trailing zeroes in binary representation of i

3GPP-MAC and XCBC

3GPP-MAC:
CBC-MAC where MAC = leftmost m bits of $E_{K'}(\Sigma)$
with $\Sigma = c_0 \oplus c_1 \oplus c_2 \oplus \cdots c_t$

Birthday attack still applies $[0, 2^{32}, 2^{33}, 0]$ if $m = 64$.

XCBC [Black-Rogaway-00]: 3-key construction
CBC with XOR of K_2 to final input block (H_{t-1}) if complete block and K_3 if padding was needed

XCBCX [Black-Rogaway-00]:
extra XOR of result with K_2/K_3 may increase strength against exhaustive key search à la DES-X

OMAC [Iwata-Kurosawa-03] renamed as **CMAC** by NIST:
derive K_2/K_3 from K (à la PMAC)

Birthday attack still applies
!! $L := E_K(0^n)$ is used by banks as key confirmation value

Impact of AES

- Exhaustive key search no longer relevant, hence MAC Algorithms 3 and 4 are not needed
- Main 'concern' for CBC-MAC variants (only exception is XOR MAC) is birthday forgery attack with 2^{64} known texts
- One but last encryption in EMAC can be dropped [Handschuh-Preneel06]

Countermeasures against birthday attack

best attacks known for AES are indicated (= conjected security level)

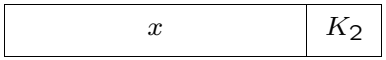
- increase the size of the internal memory n (Algorithm 5) $[0, 2^{84}, 2, 0]$ (double computation)
- randomize the first block of the data (R-MAC) [Semanko01] $[0, 2^{84}, 2, 0]$ (double MAC length)
- truncation: improves security against key recovery but adds very limited protection against birthday forgery
- prepending the length: does not work
- prepending a serial number prior to padding (with length before or after) does not work if serial numbers are in clear [Brincat-Mitchell01]: $[0, 1, 2^{65}, 0]$
- derive key in output transformation from a serial number

MAC: based on a hash function (MDC) (1)

secret prefix: $h(K_1||x)$

prepend length to avoid that one can compute $h(K_1||x||y)$ from $h(K1||x)$ without knowing K_1

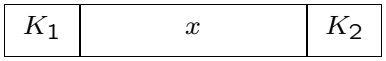
secret suffix: $h(x||K_2)$



off-line attacks on h

MAC: based on a hash function (MDC) (2)

envelope: $h(K_1||x||K_2)$



- provable security based on pseudo-randomness of compression function f
- forgery (for MD5): 2^{64} known texts
- key recovery: 2^{66} known texts and 2^{20} chosen texts

MDx-MAC (SHA-1-MAC, RIPEMD-160-MAC)

- stronger pseudo-random properties: key in each iteration
- K_1 and K_2 in separate blocks: precludes key recovery

MAC: based on a hash function (MDC) (3)

HMAC: $h_K(X) = h(K_2 \| h(K_1 \| x))$

security proof:

- h collision resistant for secret IV (*)
- f secure MAC if H_{i-1} is secret key
- need also f pseudo-random function if H_{i-1} is secret key for efficient implementation

problem: security proof no longer valid after attack by Wang et al. (*)
fixed by [Bellare06]: it is sufficient to assume that f pseudo-random function if H_{i-1} is secret key
(then HMAC is PRF and thus a secure MAC)

Attacks on NMAC and HMAC

NMAC is variant of HMAC where keys are inserted into *IV*.

[Fouqe-Leurent-Nguyen07]
NMAC MD4: 2^{88} chosen texts and 2^{95} steps

[Rechberger-Rijmen07]
NMAC MD5: key recovery 2^{45} chosen texts and 2^{100} steps (related key)
NMAC SHA-1: key recovery 34/80 rounds with 2^{155} chosen texts and 2^{157} steps (related key)
HMAC SHA-1: forgery for 37/80 rounds with 2^{65} data
HMAC SHA-1: inner key recovery for 53/80 rounds with 2^{100} data and time

Further improvements to be expected.

Dedicated MACs

Message Authenticator Algorithm (ISO 8731-2: 1987):

- D. Davies and D. Clayden [1983]
- key size $k = 64$, internal memory $m = 64$, result $n = 32$
- typical speed: 2 Mbyte/s (50% of MD4)
- special mode for messages ≥ 1024 bytes

best known attacks:

- forgery: 2^{24} messages of 1 Kbyte
- key recovery: 2^{32} chosen texts, $2^{44} \ldots 2^{51}$ multiplications
- several large classes of weak keys

Other dedicated MAC algorithms

- KHF (cryptanalyzed by Wagner)

fixed length input

- COMP-128 (GSM example)
- CAVE (CDMA)
- Secure-ID
- Two-Track-MAC

Information-theoretic authentication

authentication codes (AC)/universal hash functions
[1970s (Zobrist/Simmons/Carter-Wegman)]

- advantages
  - provably secure: only combinatorial
  - extremely fast (10-15 times faster than AES/HMAC)
  - parallelizable and incremental
- disadvantages
  - use key only once
  - sometimes very large keys: low key agility
  - security level in bits against forgery is at most half the key size

Information-theoretic authentication

[Black-Halevi-Krawczyk-Krovetz-Rogaway99]

[...] "since the combinatorial property of the universal hash-function family is mathematically proven (making no cryptographic hardness assumptions), it needs no "over-design" or "safety margin" the way a cryptographic primitive would. Quite the opposite: the [UMAC] hash-function family might as well be the fastest, simplest thing that one can prove universal."

Example: polynomial authentication code

(Change of notation: k is key rather than key size in bits)

- key k', $k \in GF(2^n)$
- split x into x_1, x_2, ..., x_t, with $x_i \in GF(2^n)$
- note $\ell = t \cdot n$

$$g(x) = k' + \sum_{i=1}^{t} x_i \cdot k^i$$

Pr(success of forgery after seeing 1 text/MAC pair) $= (\ell/n)/2^n = t/2^n$

In practice: value k can be reused

Step 1: Compress

family of functions $g_k : A \longrightarrow B$ with $a = |A|$ and $b = |B|$.
$B, \star$ an Abelian group
Let ϵ be any positive real number.

g_k is an **ϵ-almost universal** class (or $\epsilon - AU$ class) $\mathcal{G}$ of hash functions if $\forall x, x' \neq \in A$

$$\Pr_k \left\{ g_k(x) = g_k(x') \right\} \leq \epsilon \,.$$

g_k is an **ϵ-almost $\star$ universal** class (or ϵ-A$\star$U class) G of hash functions if $\forall x, x' \neq x \in A$ and $\forall \Delta \in B$

$$\Pr_k \left\{ g_k(x) = g_k(x') \star \Delta \right\} \leq \epsilon \,.$$

Step 1: Compress (2)

functions that are ϵ-AU

- $g_k(x) = \sum_{i=0}^{t} x_i \cdot k^i$ with k, $x_i \in$ GF(2^r) or GF(p)

functions that are ϵ-A$\star$U

- $g_k(x) = \sum_{i=1}^{t} x_i \cdot k^i$ with k, $x_i \in$ GF(2^r) or GF(p)
- MMH: $g_k(x) = \left(\sum_{i=1}^{t} x_i \cdot k_i\right) \bmod p$
  x_i, k_i, $\in \mathbf{Z}_{2^{32}}$ and $p = 2^{32} + 15$ (inner sum mod 2^{64}) [Halevi-Krawczyk97]
- NMH: $g_k(x) = \left(\sum_{i=1}^{t/2} (x_{2i-1} + k_{2i-1}) \cdot (x_{2i} + k_{2i})\right) \bmod p$
  x_i, $k_i \in \mathbf{Z}_{2^{32}}$ and $p = 2^{32} + 15$ [Wegman-Carter81 and Halevi-Krawczyk97]
- NH: $g_k(x) = \left(\sum_{i=1}^{t/2} ((x_{2i-1} + k_{2i-1}) \bmod 2^w) \cdot ((x_{2i} + k_{2i}) \bmod 2^w)\right) \bmod 2^{2w}$
  x_i, $k_i \in \mathbf{Z}_{2^w}$ [BHKKR99]
- WH: $g_k(x) = \left(\sum_{i=1}^{t/2} (x_{2i-1} + k_{2i-1}) \cdot (x_{2i} + k_{2i}) x^{(t/2-i)w}\right) \bmod p(x)$
  x_i, $k_i \in$ GF(2^w) (polynomials) [Kaps-Yüksel-Sunar04]

Step 2: Replace addition $k'+$

pseudorandom function family $f_{k'}$ $\rightsquigarrow$ computational security

Option 1: $\mathsf{MAC}_{k||k'}(x) = f_{k'}(g_k(x))$ with g ϵ-AU

Option 2: $\mathsf{MAC}_{k||k'}(x) = f_{k'}(n) \star g_k(x)$ with g ϵ-A$\star$U
need nonce but better security

Option 3: $\mathsf{MAC}_{k||k'}(x) = f_{k'}(n||g_k(x))$ with g ϵ-AU
need nonce and larger input of f

Observations on universal hash functions for MAC

1. reuse of k is common in practice, in particular if k is large
2. nonce n is supposed to be unique; what if it isn't?
   - nonce is always used twice (generation/verification)
   - nonce reuse for verification (e.g., if random numbers)
   - nonce reuse for MAC generation more problematic
3. weak keys: $\forall x, x', x' \neq x$ $\Pr_k\{g_k(x) = g_k(x') \star \Delta\} \leq \epsilon$ does not imply that $\forall k$ $\Pr_{x,x'}\{g_k(x) = g_k(x') \star \Delta\} \approx 1/|B|$. For some keys $\overline{k}$
$$\Pr_{x,x'}\left\{g_{\overline{k}}(x) = g_{\overline{k}}(x') \star \Delta\right\} \gg 1/|B|$$
4. simple combinatorial scheme: partial knowledge of k may be devastating (e.g., verification by leaking first 2 key bytes)
5. messages of special form reduce hash value to subspace

Related Work

[Coppersmith96] Finding a small root of a bivariate integer equation; factoring with high bits known.

[Bellare-Goldwasser97] Verifiable partial key escrow

[McGrew-Fluhrer05] Multiple forgery attacks

[Preneel-vanOorschot-Knudsen96] Key recovery attacks on ANSI retail MAC

[Blackburn-Paterson04] Cryptanalysis of a MAC due to Cary and Venkatesan

[Black-Cochran 06] Focus on reforgeability

UMAC RFC 4418 contains warnings:

- be careful if too many wrong MAC values
- "once an attempted forgery is successful, it is possible, in principle, that subsequent messages under this key may be easily forged. This is important to understand in gauging the severity of a successful forgery, **even though no such attack on UMAC is known to date.**"

Even two appendices full of warnings for GCM NIST SP 800-38D

Polynomial hash

$g_k(x) = \sum_{i=1}^{t} x_i \cdot k^i$ with k, $x_i \in \mathsf{GF}(2^n)$

GCM NIST SP-800 38-D:

- Option 2: $\mathsf{MAC}_{k||k'}(x) = \mathsf{trunc}_\tau(\mathsf{AES}_{k'}(n) \oplus g_k(x))$
- $k = \mathsf{AES}_{k'}(000\ldots00)$

trivial weak key: $k = 0$ (extremely unlikely)

trivial to verify a guess for k, even if we do not know k' (ok for GCM)

if order of k divides $l < t$: swap of two blocks leaves $g_k(x)$ unchanged

[Joux06] attack on GCM (nonce reuse by sender):
compute $\mathsf{MAC}_{k||k'}(x) \oplus \mathsf{MAC}_{k||k'}(x') = g_k(x) \oplus g_k(x') = g_k^*(x, x')$
k is one of the t roots of the polynomial g_k^*

Polynomial hash (2)

Joux: *"replacing the counter encryption for MACs by the classical encryption with the block cipher usually used with Wegman-Carter MACs seems a safe option."*

Variant on Joux attack that works even for Option 1 or 3
(more expensive, same cost as forgery, but no nonce reuse):

- obtain 1 MAC value for a text x of your choice
- choose x' such that the polynomial with coefficients from $x - x'$ has t distinct roots (each time)
- perform a MAC verification query for x'
- after $2^n/t$ trials you know that k is one of t values
- perform another t MAC verification queries to find out which one of the t

easy to take into account any information you may have on k.

Polynomial hash (3)

special message attack [Ferguson05]

- $x_i = 0$ except if $i = 2^j$ for some j: $g_k(x) = \sum_{j=0}^{l} x_{2^j} \cdot k^{2^j}$
- squaring is a linear operation in GF(2^n)
  hence we can write the bits of the hash as follows:

$$g_k(x)[.] = \sum_{i*,j*,u*} x_{i*}[j*] \cdot k[u*]$$

- choose x, x' such that $\text{trunc}_s(g_k(x)) = \text{trunc}_s(g_k(x'))$ $(1 \le s < \tau)$, independent of the value of k
- submit x' for verification (same nonce); success prob. $1/2^{\tau-s}$
- collect $r \cdot 2^{\tau-s}$ messages (same nonce!) resulting in r forgeries; each forgery yields $\tau - s$ linear equations in the key bits.

response by NIST: be careful with wrong MAC values; no error messages

MMH [Halevi-Krawczyk97]

$g_k(x) = \left(\sum_{i=1}^{t} x_i \cdot k_i\right) \bmod p$
$x_i,\ k_i, \in \mathbf{Z}_{2^{32}}$ and $p = 2^{32} + 15$ (inner sum mod 2^{64})

attacks (consider $t = 2$):

- $k_1 = k_2 = 0$: all messages map to 0.
- $k_i = 0$: result does not depend on x_i
- $k_1 = k_2$: can swap x_1 and x_2: no effect
- $k_1 = \alpha \cdot k_2 \bmod p$, $g_k(x)$ unchanged if one replaces (x_1, x_2) by $(x_2/\alpha, x_1 \cdot \alpha)$
  hence a guess for α can be verified with one verification query
- easy to exploit partial key information $x_1 k_1 + x_2 k_2 = x_1' k_1 + x_2' k_2$

NH – UMAC [BHKKR99] – VMAC – WMAC

$$g_k(x) = \left(\sum_{i=1}^{t/2} ((x_{2i-1} + k_{2i-1}) \bmod 2^w) \cdot ((x_{2i} + k_{2i}) \bmod 2^w) \right) \bmod 2^{2w},$$

$x_i,\ k_i \in \mathbf{Z}_{2^w}$

- if $k_{2i-1} = k_{2i}$: can swap x_{2i-1} and x_{2i} without changing result
- if $((x_{2i-1} + k_{2i-1}) \bmod 2^w) = 0$, $g_k(x)$ is independent of x_{2i}.
- assume that $k_{2i} = k_{2i-1} + \Delta \bmod 2^w$ for $\Delta \in \mathbf{Z}_{2^w}$,
  then $g_k(x)$ unchanged if one replaces
  (x_{2i-1}, x_{2i}) by $(x_{2i} + \Delta \bmod 2^w, x_{2i-1} - \Delta \bmod 2^w)$
- recover t key words with $t \cdot 2^w$ verification queries
- with an oracle for s bits of one key word, expected complexity to find one word reduces to 2^{w-s} verification queries.

NH – UMAC [BHKKR99] – VMAC – WMAC (2)

UMAC: $w = 32$ marginal; ok if output length of 64, 96 or 128 bits. (earlier version had $w = 16$)

VMAC: $w = 64$ (probably ok)

WMAC: variant with $w = 16$, 32 and 64

UMAC RFC 4418 [March 2006]
"It should be pointed out that once an attempted forgery is successful, it is possible, in principle, that subsequent messages under this key may be easily forged. This is important to understand in gauging the severity of a successful forgery, even though no such attack on UMAC is known to date."

Conclusions

While universal hash functions have very attractive performance and provable security, they can be very brittle in practice

- avoid reusing keys (Snow3G is good example)
- sender/verifier needs to guarantee/check uniqueness of nonces
- vulnerable to oracle that reveals part of the key (and thus to side-channel attacks)

Some schemes are more secure than others. . .

EMAC based on AES is slower but more "robust"

- internal collisions (2^{64} texts) lead to forgeries, but not to key recovery
- no known way to use an oracle that gives access to 32 or 64 key bits
- faster key setup

Ongoing: other universal hash functions, including improved attack on MAC of Cary and Venkatesan

Summary

	number of weak keys	divide and conquer attacks	partial key information
Polynomial hash GF(2^n)	1	k only	yes
Polynomial hash GF(p)	1	k only	?
MMH	type I/II	w-bit subkey k_i	yes
Square Hash	type II	w-bit subkey k_i	yes
NMH/NH/WH	type II/III	w-bit subkey k_i	yes