Hashing and sponge functions
Part 1: What we have and what we need

Joan Daemen[1]

Joint work with
Guido Bertoni[1], Michaël Peeters[2] and Gilles Van Assche[1]

[1]STMicroelectronics [2]NXP Semiconductors

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Outline

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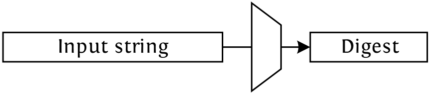
Cryptographic hash functions

- Function h
  - from any binary string $\{0,1\}^*$
  - to a fixed-size digest $\{0,1\}^n$
  - **One-way**: given $h(x)$ hard to find x…



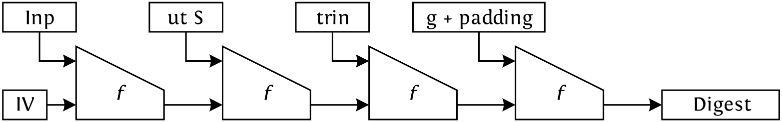
- Applications in cryptography
  - *Signatures*: $\text{sign}_{\text{RSA}}(h(M))$ instead of $\text{sign}_{\text{RSA}}(M)$
  - *Key derivation*: master key K to derived keys $(K_i = h(K \| i))$
  - *Bit commitment, predictions*: $h(\text{what I know})$
  - *Message authentication*: $h(K \| M)$
  - …

Examples of popular hash functions

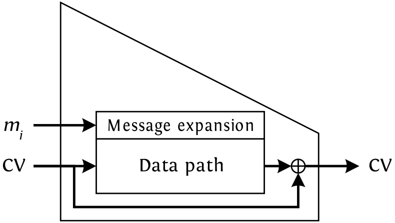
- MD5: $n = 128$
  - Published by Ron Rivest in 1992
  - Successor of MD4 (1990)
- SHA-1: $n = 160$
  - Designed by NSA, standardized by NIST in 1995
  - Successor of SHA-0 (1993)
- SHA-2: family supporting multiple lengths
  - Designed by NSA, standardized by NIST in 2001
  - 4 members named SHA-n
  - SHA-224, SHA-256, SHA-384 and SHA-512

The chaining structure: Merkle-Damgård

- Simple iterative construction:
  - iterative application of compression function (CF)
  - message length is coded in the padding
- Proven collision-resistance preserving, implying
  - CV size = digest size: *narrow pipe*
  - generating collisions for CF must be made hard



The compression function structure: Davies-Meyer



Uses a block cipher:

- Separation data path and message expansion
- Feedforward due to Merkle-Damgård

The use of basic operations

- All popular hash functions were based on ARX
  - addition modulo 2^n with $n = 32$ (and $n = 64$)
  - bitwise addition: XOR
  - bitwise shift operations, cyclic shift
  - security: “algebraically incompatible operations”
- ARX would be elegant
  - ...but silently assumes a specific integer coding
- ARX would be efficient
  - ...but only in software on CPUs with n-bit words
- ARX would have good cryptographic properties
  - but is very hard to analyze
  - ...attacks have appeared after years

Trouble in paradise

- 1991-1993: Den Boer and Bosselaers attack MD4 and MD5
- 1996: Dobbertin improves attacks on MD4 and MD5
- 1998: Chabaud and Joux attack SHA-0
- 2004: Joux et al. break SHA-0
- 2004: Wang et al. break MD5
- 2005: Lenstra et al., and Klima, make MD5 attack practical
- 2005: Wang et al. theoretically break SHA-1
- 2006: De Cannière and Rechberger further break SHA-1
- Many more results and authors
- Also generic attacks on chaining mode (see later)

Outline

A way out of the hash function crisis

- 2005-2006: trust in established hash functions was crumbling, due to
  - use of ARX
  - adoption of Merkle-Damgård
  - and SHA-2 were based on the same principles
- 2007: NIST calls for SHA-3
  - similar to AES contest
  - a case for the international cryptographic community!

SHA-3 Contest

- Open competition organized by NIST
  - NIST provides forum
  - scientific community contributes: designs, attacks, implementations, comparisons
  - NIST draws conclusions and decides
- Goal: replacement for the SHA-2 family
  - 224, 256, 384 and 512-bit output sizes
  - other output sizes are optional
- Requirements
  - security levels specified for traditional attacks
  - each submission must have
    - complete documentation, including design rationale
    - reference and optimized implementations in C

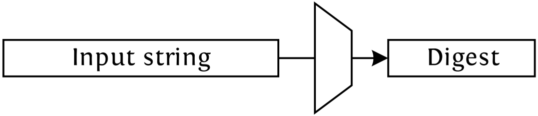
SHA-3 Time Schedule

- January 2007: initial call
- October 2008: submission deadline
- February 2009: first SHA-3 conference in Leuven
  - Presentation of 1st round candidates
- July 2009: NIST announces 2nd round candidates
- August 2010: second SHA-3 conference in Santa Barbara
  - cryptanalytic results
  - hardware and software implementation surveys
  - new applications
- December 2010: announcement of finalists
- 2012: final SHA-3 conference and selection of winner(s)

Outline

Traditional security requirements of hash functions

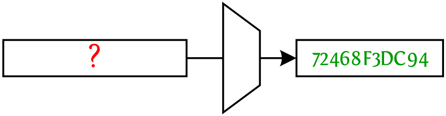
- Function *h* from $\mathbf{Z}_2^*$ to $\mathbf{Z}_2^n$



- Security requirements
  - pre-image resistance
  - 2nd pre-image resistance
  - collision resistance

Pre-image resistance

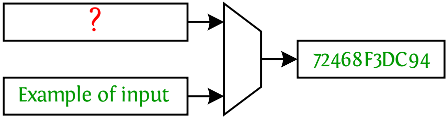
- Given $y \in \mathbf{Z}_2^n$, find $x \in \mathbf{Z}_2^*$ such that $h(x) = y$
- **Example**: given derived key $K_1 = h(K\|1)$, find master key K



- There exists a generic attack requiring about 2^n calls to h
- Requirement: there is no attack more efficient

2nd pre-image resistance

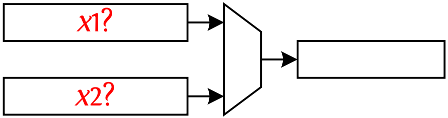
- Given $x \in \mathbf{Z}_2^*$, find $x' \neq x$ such that $h(x') = h(x)$
- **Example**: signature forging
  - given M and $\text{sign}(h(M))$, find another M' with equal signature



- There exists a generic attack requiring about 2^n calls to h

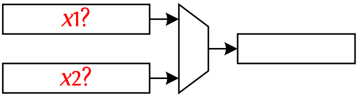
Collision resistance

- Find $x_1 \neq x_2$ such that $h(x_1) = h(x_2)$



- There exists a generic attack requiring about $2^{n/2}$ calls to h
  - Birthday paradox: among 23 people, two have the same birthday (with 50% probability)
  - Scales as $\sqrt{|\text{range}|} = 2^{n/2}$

Collision resistance (continued)



- **Example**: "secretary" signature forging
  - Set of good messages $\{M_i^{\text{good}}\}$
  - Set of bad messages $\{M_i^{\text{bad}}\}$
  - Find $h(M_i^{\text{good}}) = h(M_j^{\text{bad}})$
  - Boss signs M_i^{good}, but valid also for M_j^{bad}

Other requirements

- What if we use a hash function in other applications?
- To build a MAC function, e.g., HMAC (FIPS 198)
- To destroy algebraic structure, e.g.,
  - encryption with RSA: OAEP (PKCS #1)
  - signing with RSA: PSS (PKCS #1)
- Problem:
  - additional requirements on top of traditional ones
  - how to know what a hash function is designed for?

Contract

- Security of a concrete hash function h cannot be proven
  - sometimes reductions are possible...
  - rely on public scrutiny!
- Security claim: contract between designer and user
  - security claims $\geq$ security requirements
  - attack that invalidates claim, breaks h!
- Claims often implicit
  - e.g., the traditional security requirements are implied

List of claimed properties

- Security claims by listing desired properties
  - collision resistant
  - (2nd) pre-image resistant
  - correlation-free
  - resistant against length-extension attacks
  - chosen-target forced-prefix pre-image resistance
  - ...
- But ever-growing list of desired properties
- Moving target as new applications appear over time

But hey, the ideal hash function exists!

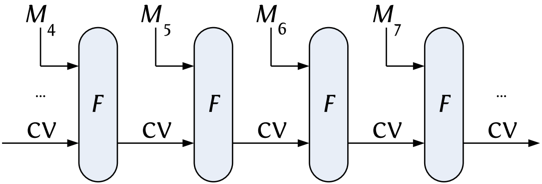
Random oracle $\mathcal{RO}$

- A random oracle [Bellare-Rogaway 1993] maps:
  - message of variable length
  - to an infinite output string
- Supports queries of following type: (M, ℓ)
  - M: message
  - ℓ: requested number of output bits
- Response Z
  - String of ℓ bits
  - Independently and identically distributed bits
  - Self-consistent: equal M give matching outputs

Compact security claim

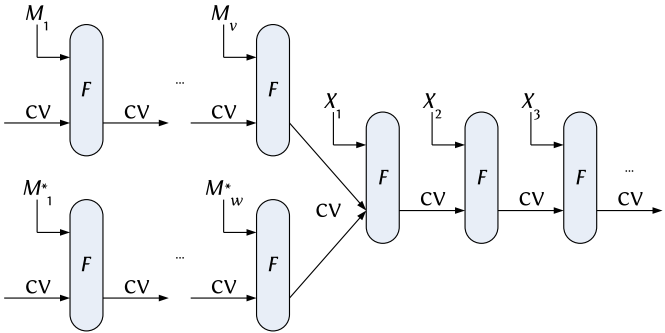
- Truncated to n bits, $\mathcal{RO}$ has all desired properties, e.g.,
  - Generating a collision: $2^{n/2}$
  - Finding a (2nd) pre-image: 2^n
  - And [my chosen requirement]: $f(n)$
- Proposal for a compact security claim:
  - "My function h behaves as a **random oracle**"
- Does not work, unfortunately

Iterated hash functions



- All practical hash functions are iterated
  - Message M cut into blocks $M_1, \ldots, M_l$
  - q-bit chaining value
- Output is function of final chaining value

Internal collisions!



- Difference inputs M and M' giving the same chaining value
- Messages $M \| X$ and $M' \| X$ always collide for any string X

How to deal with internal collisions?

- $\mathcal{RO}$ has no internal collisions
  - If truncated to n bits, it does have collisions, say M and M'
  - But $M||X$ and $M'||X$ collide only with probability 2^{-n}
  - Random oracle has “infinite memory”
- Abandon *iterated modes* to meet the $\mathcal{RO}$ ideal?
  - In-memory hashing, non-streamable hash functions?
  - Model for finite memory, internal collisions!

Variable output-length functions

- Variable-length output:
  - Single function for different hash function lengths
  - Useful, e.g., for signatures, “mask generating functions”
  - Stream cipher
- Exponential scaling of the security requirements?!?

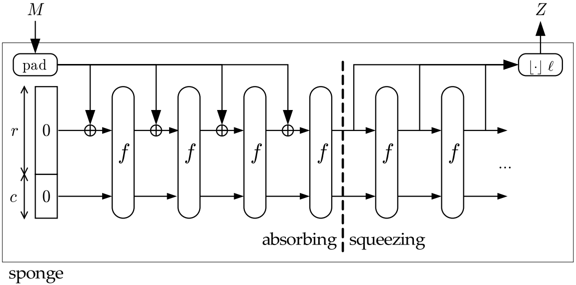
Pre-image resistance	2^n ?
2nd pre-image resistance	2^n ?
Collision resistance	$2^{n/2}$?

How to have a compact security claim?

- Try to define some *thing* Π that
  - has the same interface as $\mathcal{RO}$
  - behaves like $\mathcal{RO}$ …
  - …modulo internal collisions
- Strength of Π depends on some (size) parameters
- Compact security claim would then be:
  - "My function *h* behaves as a Π with given size parameters"
- Output length no longer appears in security claim
- What could Π be?

Outline

The sponge construction (2007)



- Calls a b-bit permutation (or transformation) f
  - r bits of *rate*
  - c bits of *capacity* (security parameter)
  - $b = r + c$
- Padding rule must satisfy some simple requirements

Random sponges

- Random T-sponge: f chosen randomly from $(2^{r+c})^{2^{r+c}}$ transformations
- Random P-sponge: f chosen randomly from $(2^{r+c})!$ permutations f
- Random sponges become our reference Π
- Express security claim now requires specifying:
  - c: capacity
  - r: bitrate
  - whether f is a permutation or transformation

Flat sponge claim

Simplifying the claim to a single parameter

Flat sponge claim with claimed capacity c

For any attack, the success probability is not above the sum of that for a $\mathcal{RO}$ and $N^2/2^{c+1}$, with N the number of calls to f

What does a flat sponge claim state?

- Example: $c = 256$
- $N^2/2^{257}$ becomes significant when $N \approx 2^{128}$
- Collision-resistance:
  - Similar to that of random oracle up to $n = 256$
  - Maximum achievable security level: 2^{128}
- (2nd) pre-image resistance:
  - Similar to that of random oracle up to $n = 128$
  - Maximum achievable security level: 2^{128}
- Flat sponge claim forms a ceiling to the security claim
  - As good as a random oracle below $2^{c/2}$ queries
  - No guarantees beyond $2^{c/2}$ queries
  - If $2^{c/2}$ is out of reach, that is OK!

Outline

The NIST SHA-3 security requirements

Output length	224	256	384	512
Collision resistance	2^{112}	2^{128}	2^{192}	2^{256}
Pre-image resistance	2^{224}	2^{256}	2^{384}	2^{512}
2nd pre-image resistance	$2^{224}/\ell$	$2^{256}/\ell$	$2^{384}/\ell$	$2^{512}/\ell$

$\ell =$ message length

- Puzzling to say the least
  - excessive requirements for (2nd) pre-image resistance
  - collisions less important than pre-images?
  - 2nd pre-image resistance of long messages less important?

The NIST SHA-3 security requirements explained

- Do not express what would be useful
- But what can hopefully be achieved by Merkle-Damgård
  - collision-resistance: that of ideal compression function
  - pre-image: appears achievable for basic case
  - 2nd pre-image: used to be 2^n but adapted after ...
- Wave of generic attacks against Merkle-Damgård
  - Joux (2004): Multicollisions
  - Kelsey and Schneier (2005): 2nd pre-image attacks
  - Kohno and Kelsey (2006): Herding attacks
  - ...
  - All use internal collisions due to narrow pipe

Outline

Conclusions

- Monoculture has resulted in hashing confidence crisis
  - ARX less secure than expected
  - Merkle-Damgård less sound than believed
- NIST calls out for help
  - addressed to the cryptographic community
  - with requirements still deeply rooted in crypto folklore
- Random sponges allow expressing compact security claims
  - express security against all thinkable attacks
  - "As good as a random oracle up to some ceiling"
  - flat sponge claim appears achievable